ECON 233 Finance 1- Week 5

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Single Index Model assumptions

- If you were to choose to create your optimal risky portfolio, taking into account the information on N assets you would have to deal with
  - estimating N returns
  - estimating $\frac{N(N-1)}{2}$ covariances
- Single index models assume that returns of all securities are driven by a single common factor.
- $r_i - r_f = \alpha_i + \beta_i(r_M - r_f) + e_i$
- Simplifies massively the covariance structure. Only need to estimate
  - N returns
  - $N + 1$ estimates of σ_i^2
- Idea- stocks move or co-move only due to the market.

β

- β_i measures the extent to which a security's return co-moves w.r.t. the overall market portfolio.
- $\beta_i = \dfrac{Cov(R_i, R_M)}{\sigma_M^2}$
- values of β
  - =1 moves one to one with market
  - $>$ 1 more volatile or moves more than one to one with market
  - $<$1 less volatile or moves less than one to one with market
  - $<$ 0 moves in the opposite direction as that of the market.

Why not only buy assets with high betas?

- Only buying assets with high β means that every asset in your portfolio moves more than one to one with the market.
- If market increases by 10% your portfolio increases by more than 10% but also vice versa.
- Susceptible to high volatility and market swings.
- Poor diversification.
- Always a good idea to have a healthy mix of high and low and even negative β assets to create a drag and lower risk.
- Can you name an example of an asset or portfolio that always moves more than 1 to 1 with the market?
- When can you justify the purchase of only that kind of asset?

Why is the R^2 value of the single index model regression the same as the proportion of systematic risk?

- Single Index Model: $R_i = \alpha_i + \beta_i R_m + \epsilon_i$
- **Variance decomposition:** $\text{Var}(R_i) = \beta_i^2 \text{Var}(R_m) + \text{Var}(\epsilon_i)$
  - $\beta_i^2 \text{Var}(R_m)$ = Systematic (market) risk
  - $\text{Var}(\epsilon_i)$ = Unsystematic (firm-specific) risk

In regression terms:

$$R^2 = \frac{\text{Explained Variance}}{\text{Total Variance}} = \frac{\beta_i^2 \text{Var}(R_m)}{\beta_i^2 \text{Var}(R_m) + \text{Var}(\epsilon_i)}$$

Therefore:

$$R^2 = \frac{\text{Systematic Risk}}{\text{Total Risk}}$$